NUCLEUS

ENGLISH FOR SCIENCE AND TECHNOLOGY

MATHEMATICS

David Hall

with

Tim Bowyer

Series Editors
Martin Bates and Tony Dudley-Evans
Science Adviser to the Series
Arthur Godman C. Chem., MRIC

Longman

Contents

Unit 8 Process 3 Cause and Effect

Section 1 Presentation

1. Read this:

Quadratic equations

A quadratic expression is an expression which contains a number raised to the power of 2 (e.g. x^2). It cannot contain numbers raised to powers greater than 2 (e.g. x^3, x^4, etc.)

Say which of the following are quadratic expressions:

a) x^2+3
b) x^2+3x+7
c) $3x^2-2$
d) x^2+x^4
e) $x+2y+z$
f) $x^3+2x-16$

2. Read this:

A quadratic expression is generally given in the form ax^2+bx+c, where x is the variable and a, b and c are constants. A quadratic equation is generally given in the form $ax^2+bx+c=0$.

Now change the following equations to the general form for quadratic equations and give the values of a, b and c. The first two are done for you:

	Given	General form	a	b	c
a)	$2x^2-3x=2$	$2x^2-3x-2=0$	2	-3	-2
b)	$3x^2=-1$	$3x^2+0x+1=0$	3	0	1
c)	$x^2=3x$				
d)	$5x-3=4x^2$				
e)	$x^2+x=1$				
f)	$5x^2+7=20$				

3. Look at this:

A quadratic equation has two solutions, called roots. If the factors of a quadratic equation can be found easily, then we can find the roots by factorising.

Example: Factorisation of $x^2+x-12=0$ gives $(x-3)(x+4)=0$. The roots of the equation are therefore 3 and -4.

Now make similar sentences about the following:

a) $x^2+7x+10=0$
b) $x^2-9x+18=0$
c) $x^2-100=0$
d) $x^2+5x-6=0$

4. Read this:

Factorisation of $x^2+12x+36$ gives $(x+6)^2$. Therefore the expression is known as a perfect square.

x^2+ax can be made into a perfect square by adding $\left(\frac{a}{2}\right)^2$.

For example, x^2+20x can be made into a perfect square by adding 100.
$x^2+20x+100$ factorises into $(x+10)^2$.

Write similar sentences about the following expressions:

a) x^2-12x
b) x^2+3x
c) x^2+7x

(**Note:** This operation is known as *completing the square*).

Section 2 Development

5. Look and read:

If the factors of a quadratic equation cannot be found easily, then we can find the roots by using the formula

$$\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

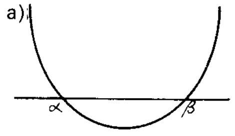
The two roots are at $\frac{-b+\sqrt{(b^2-4ac)}}{2a}$ and $\frac{-b-\sqrt{(b^2-4ac)}}{2a}$

If (b^2-4ac) is negative, then $\sqrt{(b^2-4ac)}$ is imaginary and no real roots satisfy the equation.

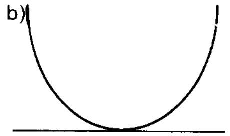
Complete these two sentences:
a) If (b^2-4ac) is positive, ...
b) If (b^2-4ac) is zero, ...

6. Look and read:

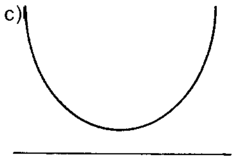
The two roots of a quadratic equation are denoted by α and β. We can show the six possible cases by drawing graphs.



- If (b^2-4ac) is positive and a is positive, then there are two real roots at α and β and the quadratic expression is only negative for values of x between α and β.

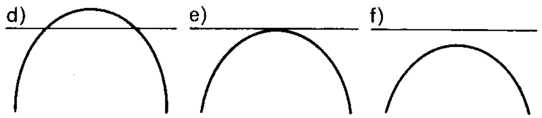


- If (b^2-4ac) is zero and a is positive, then the two real roots α and β *coincide*, and the quadratic expression is positive for all other values of x.



- If (b^2-4ac) is negative and a is positive, then there are no real roots, and the quadratic expression is always positive.

Now describe the other three cases:



7. Look at these examples:

$x^2-2x-3=0$ i) Factorisation of the left-hand side gives $(x-3)(x+1)=0$.
ii) Factorising the left-hand side gives $(x-3)(x+1)=0$.

Now change the following examples to form (ii):

a) $x^2-2x-3=0$ Addition of 4 to both sides gives a perfect square.
b) $\frac{25}{10}$ Reduction of the fraction gives $\frac{5}{2}$.
c) $9x=18y$ Division of both sides by 9 gives $x=2y$.
d) $x^2+10x+32$ Subtraction of 7 from this expression gives a perfect square.
e) $x^2-5x+6=0$ Solution of this equation gives roots at 2 and 3.
f) $a-\frac{a}{x^2}=0$ Multiplication of both sides by x^2 gives $ax^2-a=0$.

8. Look at this example:

$x^2-10x-200=0$.
Factorising, we obtain $(x+10)(x-20)=0$

Use expressions from this list to complete the calculation below.

completing the square,
dividing ......,
factorising,
subtracting ......,
subtracting ......,
taking the square root ......,

Given $ax^2+bx+c=0$

...... we obtain $x^2+\frac{b}{a}x+\frac{c}{a}=0$.

...... we obtain $x^2+\frac{b}{a}x=-\frac{c}{a}$

...... we obtain $x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

...... we obtain $\left(x+\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

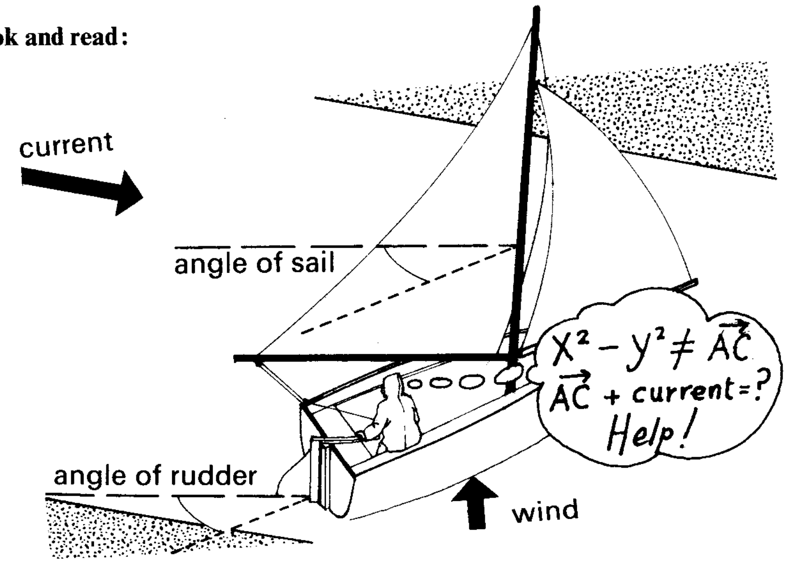
...... we obtain $x+\frac{b}{2a}=\frac{\pm\sqrt{(b^2-4ac)}}{2a}$

...... we obtain $x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$

This gives the formula for finding the roots of a quadratic equation.

Section 3 Reading

9. Look and read:



Addition of vectors

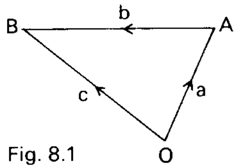


Fig. 8.1

Taking O as the origin, we can determine any position A by naming the length and direction of the line which joins O to A. Thus the vector a in Figure 8.1 is denoted by $\overline{OA}$ or $\overrightarrow{OA}$.

We may reach a second position, B, by displacement first from O to A and then from A to B. But direct displacement from O to B gives the vector $\overline{OB}$. By comparing these two results, we see that $\overline{OA}+\overline{AB} = \overline{OB}$(1). If we complete the parallelogram OABC, then $\overline{CB} = \overline{OA}$, and $\overline{OC} = \overline{AB}$.

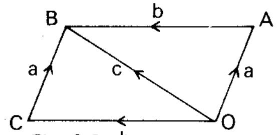
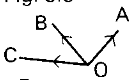


Fig. 8.2

Replacing $\overline{AB}$ by $\overline{OC}$ in equation (1) gives $\overline{OA}+\overline{OC} = \overline{OB}$ (2). This gives the parallelogram law for the addition of two vectors. $\overline{OB}$ is known as the resultant of $\overline{OC}$ and $\overline{OA}$.

We can repeat this addition as often as we like so that we can find the resultant of any number of vectors.

Fig. 8.3



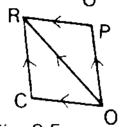
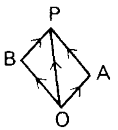


Fig. 8.5

Fig. 8.4



If we want to find the resultant of the three vectors $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ (Fig. 8.3), then we first add $\overline{OA}$ and $\overline{OB}$ giving the resultant $\overline{OP}$ (Fig. 8.4). Adding the resultant $\overline{OP}$ and the third vector $\overline{OC}$ (Fig. 8.5), we have $\overline{OA}+\overline{OB}+\overline{OC} = \overline{OR}$.

10. Say whether the following statements are true or false. Correct the false statements.

a) In Figure 8.1, subtracting AB from OB gives AO.
b) Equation (1) means that two sides of a triangle can be equal to the third side.
c) By addition of $\overline{BC}$ and $\overline{OA}$ in Figure 8.2, we obtain $\overline{AC}$.
d) Taking the vectors $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ (Fig. 8.3) in a different order gives a different resultant.
e) In Figure 8.2, a + b − c = zero.

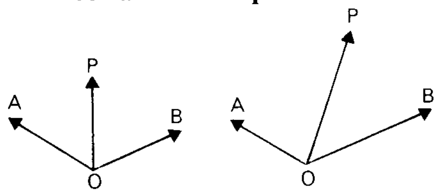
11. Look at these sentences and put them in the correct order:

They are taken from a paragraph of instructions for constructing the resultant of OA and OB in Figure 8.3.

a) Join OP.
b) Then, with radius OB, draw an arc with centre A.

c) $\overline{OP}$ is the desired resultant.
d) The second arc cuts the first at P.
e) With radius OA, draw an arc with centre B.

12. Look at this example:



The effect of making $\overline{OB}$ longer is to make $\overline{OP}$ longer.

Now write similar sentences about the following:

a) The effect on parallelogram OAPB of making $\overline{OA}$ have the same magnitude as $\overline{OB}$.
b) The effect on angles AOP and BOP of making $\overline{OA}$ have the same magnitude as $\overline{OB}$.
c) The effect on parallelogram OAPB of making $\overline{OA} = \overline{OB}$.
d) The effect on the resultant of adding $\overline{OA}$ and $\overline{BO}$.

Section 4 Listening

Multiplication of vectors by scalars

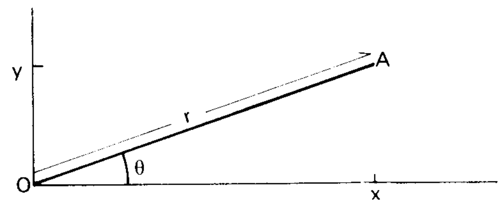


Fig. 8.6

13. Listen to the passage and write down these words in the order in which you hear them:

either	opposite	polar
alter	unchanged	process
negative	multiply	way

14. Listen again and complete this exercise:

a) Sketch a Cartesian co-ordinate system showing the effect of multiplying $\overline{OA}$ in Figure 8.6 by 3.
b) What are the polar co-ordinates of the resultant vector?
c) What are the rectangular co-ordinates of the resultant vector?
d) If $\overline{OA}$ is multiplied by a negative scalar quantity, in which quadrant is the resultant vector?
e) What are the rectangular co-ordinates of any vector (x, y) multiplied by −a?

15. Look at this example:

Adding two vectors with different directions produces a change in direction.

Now make similar sentences about the following:

a) Multiplying a vector by a scalar ...... (magnitude)
b) Multiplying a vector by a scalar ...... (direction)
c) Multiplying a vector by a negative scalar ...... (angle)
d) Multiplying a vector by a negative scalar ...... (direction)

16. PUZZLE

Comment on the following:

Given: $a > b$

c is the arithmetic mean of a and b.

Therefore $c = \frac{a+b}{2}$

or $a+b = 2c$

Multiplying both sides by $a-b$, we obtain

$$(a+b)(a-b) = 2ac-2bc$$

i.e. $a^2-b^2 = 2ac-2bc$

Adding b^2 to both sides gives $a^2 = 2ac-2bc+b^2$
Adding c^2 to both sides gives $a^2+c^2 = 2ac-2bc+b^2+c^2$
Subtracting 2ac from both sides gives $a^2-2ac+c^2 = b^2-2bc+c^2$
Factorisation of both sides produces $(a-c)^2 = (b-c)^2$
Taking the square root, $a-c = b-c$
Adding c to both sides $a = b$